The Kalpana One Orbital Space Settlement Revised

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“For me the single overarching goal of human space flight is the human settlement of the solar system, and eventually beyond. I can think of no lesser purpose sufficient to justify the difficulty of the enterprise, and no greater purpose is possible.” Mike Griffin, NASA Administrator.

I. Abstract

We present a revision of the Kalpana One orbital settlement design.1, 2 The new design fixes a rotational stability problem, which shrinks the settlement so the new population target is 3,000 residents. Kalpana One is intended to improve on the space settlement designs of the mid-1970s: the Bernal Sphere, Stanford Torus, and O’Neill Cylinders, as well as on Lewis One, designed at NASA Ames Research Center in the early 1990s. These systems are intended to provide permanent homes for communities of thousands of people. The Kalpana One structure is a cylinder with a radius of 250m and a length of 325m. Cylinders minimize shielding mass per unit of 1g living area compared with other feasible shapes. Radiation shielding dominates the mass of most space settlement designs. The radius is the minimum necessary to provide 1g at the hull when rotating at no more than 15rpm. The length is the longest possible while ensuring rotational stability.

Kalpana One’s axis of rotation is aligned with the solar system’s north-south axis to provide continuous natural light through transparent end caps. Wobble control is provided by weights attached to cables on motorized winches under computer control. Exterior maintenance is by teleoperated, semi-autonomous robots. Up to ten tons of lunar/NEO regolith radiation shielding per square meter is placed inside the hull requiring greater hull strength relative to older designs but eliminating a major failure mode. Emergency power is provided by body-mounted solar cells, but primary power comes from solar power satellites beaming energy to a body-mounted rectenna. Thermal rejection is provided by a thermal array disk. The 1g living area in the hull is supplemented by internal cylinders at lower g-levels for industry, storage, agriculture, retirement communities and recreation.

Although the design is not orbit-specific and is intended to be replicated many times, and expanded, the first Kalpana One orbital settlement may be built in an equatorial Low Earth Orbit (LEO) at an altitude of approximately 600 km or so; high enough to avoid rapid entry into the Earth’s atmosphere and minimize reboost requirements, but low enough for the van Allen Belts to provide radiation protection to reduce shielding mass. Since providing sufficient materials is one of, if not the, most difficult part of building the first orbital space settlement, it is hoped that the Kalpana One design and initial location will bring settlement of the cosmos a bit closer to reality.

II. Introduction

Although humanity has always lived on Earth, mankind is space-faring and, as the great Russian visionary Konstantin Tsiolkovsky said, “Earth is the cradle of Mankind, but one cannot stay in the cradle forever.” In the 1970s, Princeton physicist Gerard O’Neill led two Stanford/NASA Ames Research Center summer studies that supported the feasibility of kilometer-scale orbital cities.3, 4 These studies assumed that the NASA space shuttle would operate as expected, a flight every week or two, $500/lb. to orbit, and one failure per 100,000 flights. The studies also assumed that a more efficient follow-on heavy lift launcher would be

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developed. When the shuttle missed its design goals by an order of magnitude or more, it became apparent that there was no transportation system capable of supporting space settlement activities and interest waned.

The activities of the 1970s produced three space settlement designs, examples of three of the four feasible shapes for orbital settlements that rotate to provide pseudo-gravity: the Bernal Sphere, Stanford Torus, and O’Neill Cylinders. All of these settlements featured natural sunlight directed into the settlement by mirrors and rotation rates and radii consistent with 1g pseudo-gravity. The first two have passive radiation shielding in non-rotating structures, the O’Neill Cylinders have passive shielding that rotates. Note that dumbbell shaped settlements are also feasible but the authors are not aware of any well-articulated designs in the literature.

Even after it became generally known that the shuttle would not meet expectations, low level space settlement technical activities continued. The Space Studies Institute, founded by Dr. O’Neill, funded research activities and held a bi-annual conference at Princeton until the early 2000s. The Lewis One cylindrical space settlement design was presented at one of these. Lewis One abandoned natural light so that agriculture could be conducted in smaller-radius cylinders at lower g levels, reducing the size of the hull and the required shielding mass. Eliminating the complex mirror geometry simplified Lewis One relative to the earlier designs. Like all the early settlements, Lewis One was woefully under-powered as insufficient area was allocated to solar energy collection. Also, Lewis One kept the rotating habitat a few meters from non-rotating shielding. Contact between these is a catastrophic failure mode not addressed by any of the studies. This is only one of many engineering problems between today and the opening of the first space settlement.

Taken together, the earlier designs have a number of serious problems:

1. Excessive shielding mass (Bernal Sphere, Stanford Torus)
2. Extremely large mirrors to bring in natural sunlight (O’Neill Cylinders)
3. Lack of natural sunlight (Lewis One)
4. Rotational instability (Bernal Sphere, O’Neill Cylinders)
5. Lack of wobble control (Bernal Sphere, Lewis One, O’Neill Cylinders, Stanford Torus)
6. Catastrophic failure modes due to rotating hulls with minimal clearance to non-rotating shield mass (Lewis One, Stanford Torus, Bernal Sphere)

Orbital settlements, of course, are only one potential target for early space settlement. Indeed, Earth’s Moon and Mars are usually considered better locations. While we are accustomed to living on the outside of large solid spheres and these bodies provide easy access to materials, there are substantial reasons why Earth orbiting settlements will come first. Specifically,

1. 1g (pseudo-)gravity levels are possible on orbital settlements (vs.1/3 (Mars) - 1/6g (Moon)); this is critical for raising strong children
2. Rapid resupply from Earth
3. Continuous, ample, reliable solar energy
4. Better communication with Earth
5. Great views of Earth (and eventually other planets)
6. Weightless and low-g recreation near the axis of rotation
7. Relatively easy 0g construction of large living structures
8. Greater independence
9. Much greater growth potential
10. Near-Earth orbital settlements can service our planet’s tourist, exotic materials and energy markets more easily than the Moon, and Mars is too far away to easily trade with Earth.
Figure 1. Kalpana One, a 250m radius, 325m length cylindrical orbital space settlement.
Figure 2. Earlier settlement designs.
Figure 3. NASA Ames’ Lewis One orbital settlement design.
The materials supply problems can be overcome, with some difficulty, by transporting materials from the Moon and Near Earth Objects (NEOs). It should be noted that no one location on the Moon or Mars is likely to have everything required, and substantial materials transport problems will be encountered there as well.

The next section describes the exterior morphology of Kalpana One and the reasons behind these choices. In the process, the controversy over the best shape, torus, sphere, or cylinder, is resolved in favor of the cylinder. For a reasonably sized settlement, a cylinder requires hundreds of thousands of tons less radiation shielding. The following section describes the interior morphology of the Kalpana One, in particular mechanisms to take advantage of the large interior space provided by a cylinder. This is followed by a section on power and thermal considerations.

III. Hull Morphology

The primary point of space settlement is to provide living area for human beings, preferably very high quality living area. An unprotected human in high orbit (above the van Allen Belts) cannot survive naturally occurring radiation for long periods of time. Not only will periodic solar events generate sufficient radiation to kill in a few hours, ubiquitous radiation of cosmic origin degrades biological tissue continuously. Adequate radiation protection, outside of solar flare events, may be provided by approximately 4.5 tons of material per square meter of hull surface. However, Johnson based this figure on a 0.5 rad dose per year which is appropriate for the general population. As children will be born and raised in Kalpana One, and the fetus and infants are particularly vulnerable to radiation, more shielding will likely be required.

Ideally, the radiation environment inside a space settlement would be no more severe than that on Earth. The Earth is protected by the van Allen Belts and approximately 10 tons/m² of atmosphere. Thus, in the worst case, a settlement below the van Allen Belts requires 10 tons/m² of shielding, although closer analysis and careful choice of materials may reduce this figure considerably. On the other hand, settlements located above the van Allen Belts are subject to a much higher flux of cosmic radiation and are vulnerable to solar flares, which may substantially increase shielding requirements. In any case, regardless of the actual levels, settlements near L5 or in 2-1 resonance orbits as have been proposed will require far greater shielding than in LEO.

All materials must be imported from the Moon or NEOs (Near Earth Objects - asteroids and comets orbiting the Sun near Earth), and this is a very tall pole in space settlement design. It is thus necessary to choose a shape that minimizes the hullSurfaceArea/1gLivingArea, where 1gLivingArea is area available for people to live; assuming the first settlers are not willing to perform a major, uncontrolled experiment on their children’s physical development; an experiment with a high probability of causing serious problems, i.e., children with very weak bones and muscles from growing up in << 1g. Without children, you don’t have a settlement, so 1g is a hard requirement for at least a few generations.

For children to grow up with normal strength, orbital settlements must provide a pseudo-gravity environment consistent with human experience over evolutionary time. In other words, residents in the primary living area must experience an acceleration of approximately 1g (9.8m/s). This can be accomplished by rotating the settlement. Thus, orbital settlements must be rotationally symmetric around at least one axis. This limits the practical shapes to the sphere, torus, dumbbell and cylinder. Of these, the cylinder minimizes hullSurfaceArea/1gLivingArea and thus radiation shielding mass. Consider:

1. Torus. The maximum hullSurfaceArea/1gLivingArea corresponds to living area that intersects the center of the minor diameter. This means that hullSurfaceArea/1gLivingArea = \( \frac{2\pi r}{2r} \geq \pi = 3.1415... \)

2. Sphere. The surface area is \( 4\pi r^2 \) \( (r = \text{radius}) \). The living area is a function of where the flat surfaces are placed. In the worst case this is one line corresponding to the maximum distance from the axis of rotation making hullSurfaceArea/1gLivingArea infinite. To expand the living area, it must be brought closer to the axis of rotation, which simply yields a cylinder, albeit with more shielding area than necessary.

3. Dumbbell. The surface area is a function of the width of the arms and the size of the expanded area, but for all cases it is larger than for the other shapes.
4. Cylinder. For an infinitely long cylinder $\frac{\text{hullSurfaceArea}}{\text{1gLivingArea}} = 1$, but long cylinders are rotationally unstable. For reasons described below, the maximum length of a rotationally stable cylinder is approximately $1.3r$, which leads to $\frac{\text{hullSurfaceArea}}{\text{1gLivingArea}} \approx 1.77$ (note: earlier papers incorrectly listed these figures as 2.2 and 1.45 respectively).

5. Double cylinder. Consider a torus with a square cross section rather than a circular one, or, alternately, two cylinders with the same center axis but slightly different radii. The $\frac{\text{hullSurfaceArea}}{\text{1gLivingArea}} > 2$ if the two radii are nearly equal and the $\frac{\text{hullSurfaceArea}}{\text{1gLivingArea}}$ approximates a cylinder as the interior radius approaches zero. As these objects can have a larger height than cylinders due to rotational stability considerations, there may be certain parameters where they are competitive with cylinders. Alternately, hybrid designs may make sense.

Nonetheless, the best shape for an orbital space settlement appears to be a cylinder because, for any given $\text{1gLivingArea}$, the total mass is substantially less than the nearest competitor. As a typical settlement design has a mass of millions of tons and total mass is quite likely the key driver, this is a killer trade, meaning that this issue is so important there is no need for a more detailed comparison. As an extra bonus, the cylinder provides the largest interior volume per unit shielding mass (except for the rotationally unstable sphere), a factor that can be put to good use as we will see below.

The size and shape of a cylindrical settlement is determined by the radius, length, and the nature of the end caps. As Kalpana One is intended to be the first settlement built, it should be as small as practical since the shear size of a settlement is a major system driver. The minimum radius is determined by the desired pseudo-gravity level ($9.8m/s^2$) and the maximum rotation rate consistent with human needs. We assume a maximum rotation rate of $2\text{rpm}$, so the radius must be approximately $250m$. Note that the $2\text{rpm}$ figure is not well supported and further research, preferably in orbit, will be necessary to refine it.

In an ideal space environment, any cylinder rotating about its longitudinal axis will continue to do so forever; but in the real space environment perturbations cause rotating systems to eventually rotate about the axis with the greatest angular moment of inertia. If that axis is not along the cylinder length, this introduces a catastrophic failure mode where the settlement gradually changes its rotational axis until it is tumbling end-over-end. This would be the fate of unpaired O'Neill Cylinders without active controls, and passive control is always preferred. To achieve passive rotational axis-of-rotation stabilization, assuming the hull has constant mass per unit surface area, the maximum length of a cylinder is determined by its radius. Experience with spin-stabilized spacecraft suggests that the desired axis of rotation should have an angular moment of inertia at least 1.2 times greater than any other axis. For a flat-capped cylinder $a$, this means the length must be less than $1.3r$ (see appendix for details). This leads to $\frac{\text{hullSurfaceArea}}{\text{1gLivingArea}} = 1.45$ for cylinders. Thus, Kalpana One’s $\text{1gLivingArea}$ is approximately $1570m$ by $325$, for a total of $510,000m^2$, providing $170m^2$ living area for each of 3,000 residents. This is slightly more than the 155.2 required by Johnson but considerably in excess of the 98.3 per resident of 1970s New York City. Furthermore, in Kalpana One substantial additional area is available in the inner cylinders (see below). The total size is a bit smaller than some very nice California beach towns with around 10,000 residents. This is important, because above all space settlements must be nice places to live. Otherwise, even if settlers can be convinced to move in, they will leave after a short stay.

Ten tons of imported matter per square meter on the inside of the hull for radiation shield implies a total mass for Kalpana One of perhaps seven million tons. Rotating the shielding requires a stronger hull than earlier designs, but avoids catastrophic failure modes when a rotating habitat contacts non-rotating shielding only a few meters away. Shielding on the interior doubles as soil for plants. Along with air pressure matching Shimla, a large high-altitude city in India, the radiation shielding establishes the strength requirements for the hull. For omnidirectional radiation sources, i.e., cosmic radiation, radiation is minimized just inside the hull. This is because, near the hull, radiation from an essentially infinitely distant source from some directions passes through the shielding at an angle and is thus more likely to be absorbed. In the center of the settlement, all cosmic radiation passes through a minimal amount of shielding. This means that just inside the hull provides the most Earth-like living environment (1g pseudo-gravity; soil for plants, and minimal radiation).

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*aKalpana One does not have flat end caps, but only the flat section is shielded. The protruding end caps are a lightweight material to redirect sunlight into the settlement and are not used for day-to-day living, only docking operations. The shielded flat end caps must be of a transparent material or use a mirrored chevron design to bring light into the settlement.*

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Although Kalpana One is rotationally stable, it will tend to wobble since the mass distribution is unlikely to be perfectly uniform and people, machines, and materials will be in constant motion. Thus, active control will be necessary to maintain smooth rotation and avoid the equivalent of earthquakes. This may be accomplished by placing large weights attached to cables controlled by motorized, computer controlled winches on the exterior of the hull. The cables can be let out and brought in to compensate for changes in mass distribution based on data from accelerometers placed about the hull. The size, distribution, and control algorithms for this system would be an excellent thesis topic.

To complete a description of the Kalpana One hull morphology we must determine the nature of the end caps. End cap shielding should be flat to minimize mass, but a light-weight curved outer cap can be added. Surprisingly, this provides an opportunity to provide natural sunlight without appendages. Kalpana One’s rotational axis is aligned with the solar system’s north-south axis, so sunlight falls on half of each end cap continuously. If the end caps are designed to let this light in, not let too much of it out, and either reflect or diffuse the light into the interior, then Kalpana One will enjoy continuous natural sunlight. While most of humanity is accustomed to a 24 hour day/night cycle, people in the extreme northern and southern latitudes, such as Alaska, southern Chile, and the Scandinavian countries, have lived with continuous sunlight for months at a time for thousands of years. To choose the exact shape of the Kalpana One end caps will require a detailed analysis of lighting requirements and the properties of the feasible materials and coatings. While an opaque strong material readily available from lunar or NEO materials should be used for most of the hull, the end caps need to be either transparent with coatings to reflect light into the interior, must diffuse sunlight, or implement some combination.

All systems require maintenance, and Kalpana One’s exterior will be no exception. Astronauts working on the hull exterior would experience > 1g centrifugal acceleration away from the hull. This is an unacceptable risk, so all external maintenance must be accomplished by teleoperated or automated robots. For mobility, we propose single wheeled robots. The single wheel fits in gaps between body-mounted reinforced solar panels. These gaps are sized such that the wheel cannot be forced through. Thus, with a sufficiently strong connection robots cannot be accidentally thrown into space. The reinforced solar panels double as micro-meteoroid bumpers.

IV. Interior Morphology

Cylinders have only a 11.5% hullSurfaceArea/1gLivingArea advantage over the double cylinder. However, single cylinders have a much larger volume to hull surface area ratio. To take advantage of this feature, we propose placing nested smaller-radii cylinders inside Kalpana One’s hull. This will have the added advantage of providing some vertical privacy for residents at the hull. The largest of the interior cylinders should have a radius on the order of 100m less than the hull to provide ample head room in the 1g living area. The number and spacing of interior cylinders can be varied to meet the needs of each settlement. A great deal of area is available, far more than at the hull. For example, seven internal cylinders starting at a 140m radius at 20 meter spacing provides approximately 1,142,000m², over twice the space available at the hull. This is important since one of the primary disincentives to space settlement life is restricted living space. This figure can be easily increased by simply using more cylinders more tightly spaced.

The interior cylinders can be attached to the hull and each other by tension cables running from the innermost level to the outer hull. Cables passing completely through the interior can reinforce the hull and reduce hull material strength requirements. These cables may double as scaffolding for low-light-level vines, such as those found in tropical rain forests, to create a unique and beautiful interior. The presence of large numbers of vines can help keep the air clean and provide potable water via transpiration. Ideally, these vines might also provide food, but that may be too much to ask for the natural light levels available inside Kalpana One.

For transition between cylinders, elevators and ramps may be used. Ramps provide transport for heavy industrial and agricultural goods. Long trailing vines may be cultivated on the edges of the ramps to complete a scene perhaps reminiscent of the famous hanging gardens of Babylon. This is no mere fluff. To attract suitable colonists, who will need to be technically capable and therefore fairly well off, space settlements must be attractive, wonderful places to live. Feeling noble about settling the solar system will last until about the 30th diaper change, but no longer. Practical space settlements must take advantage of positive characteristics of space life that cannot be replicated on Earth.

One attraction of orbital living is low-g recreation. As most of the interior cylinders will rotate at the
same rate as the hull (2rpm), the pseudo-gravity level will be less. Thus, the internal cylinders are ideal for low-g sports, dance, and other entertainment as well as industrial activities with minimal out-gassing. In a small, closed environment such as Kalpana One air pollution is absolutely unacceptable. Low-g internal cylinders are also ideal living areas for the old and infirm who may prefer low-g living to wheelchairs and walkers.

Inside the inner-most cylinder are 0g recreation areas. Besides open space for 0g sports, dance and general play, e.g., human powered flight, Kalpana One provides a cylindrical swimming pool and 0g hotel rooms for tourists and residents. Cylindrical swimming pools were proposed by Heppenheimer. Since the swimming pool wraps around the axis of rotation, one can swim continuously without turns and dives straight up are possible. To provide sufficient pseudo-gravity to keep the water in the pool at a 20 – 40m radius, the pool may be spun at greater than 2rpm and maintained in place by electro-magnetic bearings. These bearings are similar to the levitation and propulsion systems used in maglev trains, but with much lower performance requirements. In addition to recreation, water is an excellent radiation shield, so the swimming pool can double as a solar storm shelter. For 0g hotel rooms, the 2rpm rotation rate is an irritation. People and objects will tend to collect on one wall. Hotel rooms may be despun and maintained on electro-magnetic bearings similar to those used for the swimming pool.

High intensity, controlled environment agriculture requires 50m² to feed one person. For a population of 3,000, the total agriculture area required is 150,000m². This requirement can be easily satisfied by the 140m radius internal cylinder alone, assuming that sufficient food crop species are insensitive to lower gravity levels. In fact, low-g agriculture may be more efficient than 1g since species can be bred with weaker trunks and stems leaving more energy available for edible portions of the plant. The agricultural area may be divided into a number of chambers, each of which grows a particular species. Each chamber may be sized to provide one or a few day’s need. These chambers can be operated under controlled atmosphere, temperature and lighting conditions for rapid, efficient growth of crops. Plants also clean the air and provide clean water through transpiration. However, it may be difficult to bring direct sunlight onto the entire outermost internal cylinder, so Kalpana One’s agriculture requires artificial lights which, in turn, require a great deal of power.

V. Power and Thermal Control

Small amounts of emergency power can be supplied by body-mounted solar cells. However, Kalpana One requires substantial energy resources. Approximately 60kW continuous energy per resident is required, 50kW for intensive artificial light agriculture and 10kW for other purposes. The 10kW figure reflects total energy use per person in the U.S. today, including industrial use. For a population of 3,000, this implies 180MW continuous power. We propose using separate solar power satellites (SPS) for primary energy needs. The hull exterior then requires body-mounted microwave rectennas to receive energy from solar power satellites. Wireless transmission of electrical power has been demonstrated with > 90 percent efficiency. This implies that the microwave power density must approach 390w/m² on the hull.

The heat generated by electric power consumption and incoming sunlight must be dissipated. Kalpana One's thermal rejection system consists of thermal radiators attached around the rotation axis of the settlement in the middle as shown in figure 1. Placement in the middle further enhances rotational stability. Since the rotation axis is always normal to the sunward vector, short shades around the radiators are sufficient to avoid thermal interaction with the sun. The required surface area is determined by electrical power and solar lighting inputs. A disk with 560m radius outward from the hull appears adequate assuming the thermal rejection capacity of the International Space Station thermal rejection panels, 1kW for a 12 x 4 ft panel.

VI. Future Work

There are literally thousands of questions which must be answered before design and construction of Kalpana One can begin. We are focusing on two of these in the near term:

1. Can Kalpana One be located beneath the van Allen belts and still have a passive orbital lifetime sufficient to respond should all control be lost? A computer program to simulate settlement orbital decay is under development and preliminary data suggest that the answer is yes. However, a micro-satellite mission to gather more detailed data on the inner van Allen Belt may be required.

— it may be possible to accomplish this using light pipes at considerable energy savings
2. How can the material needs of Kalpana One be supplied from extra-terrestrial resources? Here we are focusing on an electric-propulsion follow-on to the AsterAnts concept.\footnote{Arora, N., Bajoria, A., and Globus, A., “Kalpana One: Analysis and Design of a Space Colony,” 47th Structures, Structural Dynamics and Materials Conference, No. AIAA-2006-2183, AIAA, ASME, ASCE, AHS, ASC, May 2006.} AsterAnts proposes a large number of nearly identical missions to retrieve small asteroids whole. The original concept used solar sails, which present some severe problems.

VII. Conclusions

Kalpana One is intended to be the first, and smallest, of a family of space settlements. The size is determined by the limited rotation rate humans are assumed to tolerate, $2\text{rpm}$. The rotation rate drives the radius to achieve 1g pseudo-gravity, and the radius drives the length due to angular moment of inertia requirements. For later, larger settlements in the Kalpana family, the rotation rate may be reduced, increasing the radius and the allowable length.

Kalpana One solves some of the problems found in earlier designs: excessive shielding mass, large appendages, lack of natural sunlight, rotational instability, lack of wobble control, and some catastrophic failure modes. Much is left to be done before a practical space settlement can be fully designed and built. Just as our distant ancestors left the warm oceans and colonized dry land, it is our task to settle the vast, empty reaches of space; thereby ensuring the survival and growth of civilization, humanity, and life itself. Let’s get to work.

VIII. Acknowledgements

The Kalpana family of space settlement designs is named in honor of Dr. Kalpana Chawla, who died when the space shuttle Columbia broke up returning from orbit \footnote{Johnson, R. and Holbrow, C., “Space Settlements: A Design Study,” Tech. Rep. SP-413, NASA, 1975.}. Before joining the astronaut corps, Dr. Chawla worked at NASA Ames Research Center in an office next door to the first author. She shared her country of origin with the other two. It is our hope that this design will evolve into the first space settlement built, and retain its name. We would like to thank the Priya Education Society, India for sponsoring this project.

References


\footnote{It should be noted that this name was first used by a team from New Delhi, India, in their submission to the NASA Ames Student Space Settlement Design Contest in 2005.}
IX. Appendix: Rotational Stable Cylinders

A. Cylindrical Shell, Sans Endcaps

First, consider just the cylindrical shell, without endcaps. The thickness is much less than the radius, so this can be modeled as a thin shell, which has a moment of inertia along the longitudinal axis ($I_z$) of:

$$I_z = Mr^2$$  \hspace{1cm} (1)

where $M$ is the mass of the shell and $r$ is its radius. Along the other two axes, the moment of inertia is:

$$I_x = I_y = 1/2Mr^2 + 1/12Mh^2$$  \hspace{1cm} (2)

where $h$ is the length (i.e. height) of the cylinder. To be stable, we want

$$I_z > 1.2I_x$$  \hspace{1cm} (3)

Substituting the equations above and solving for $h$ yields:

$$h < 2r$$  \hspace{1cm} (4)

So, a thin cylindrical shell (without endcaps) is rotationally stable as long as its length is less than or equal to its diameter.

B. Cylindrical Shell, Flat Endcaps

Now consider endcaps. The best case, in terms of stability, would be flat endcaps; anything else is going to move more mass away from the $X$ and $Y$ axes, while not moving any further away from the $Z$ axis, and thus make stability worse (except perhaps concave endcaps, which may make sense but are not considered here).

What mass to assume for the endcaps? We want them to be the same density and thickness as the cylindrical shell. So, the ratio of shell-to-cap masses will be the same as the ratio of their areas:

$$M_{cap}/M_{shell} = A_{cap}/A_{shell}$$  \hspace{1cm} (5)

Substituting the area formulas for disk and cylinder, and solving for $M_{cap}$, gives us:

$$M_{cap} = M_{shell}r/(2h)$$  \hspace{1cm} (6)

The moments of inertia for a disk rotating about its centroid are:

$$I_z = 1/2Mr^2$$  \hspace{1cm} (7)

$$I_x = I_y = 1/4Mr^2$$  \hspace{1cm} (8)

But using the parallel axis theorem, we can adjust these for rotating about the cylinder centroid, and write the mass in terms of cylinder mass from the definition of $M_{cap}$ above, giving the following for one cylinder endcap:

$$I_z = 1/2(M_{shell}r/2h)r^2$$  \hspace{1cm} (9)

$$I_x = I_y = 1/4(M_{shell}r/2h)r^2 + (M_{shell}r/2h)(h/2)^2$$  \hspace{1cm} (10)

Simplifying, and using $M$ again to refer to the mass of the cylindrical shell:

$$I_z = Mr^3/(4h)$$  \hspace{1cm} (11)

$$I_x = Mr^3/(8h) + Mrh/8$$  \hspace{1cm} (12)

So, for a total system of a cylindrical shell plus two flat endcaps, we have:

$$I_z = Mr^2 + 2(Mr^3/(4h))$$  \hspace{1cm} (13)
\[ I_x = 1/2Mr^2 + 1/12Mh^2 + 2(Mr^3/(8h) + Mrh/8) \]  

(14)

Simplifying:

\[ I_z = Mr^2(1 + r/2h) \]  

(15)

\[ I_x = M(r^2/2 + h^2/12 + r^3/(4h) + rh/4) \]  

(16)

So, the flat endcaps have a substantial and detrimental effect on stability. For example, when \( r = 100 \) and \( h = 200 \), \( I_z/I_x = 0.86 \), rather than the 1.2 it would be without the endcaps.

Numerically, we’ve found that \( I_z/I_x = 1.2 \) when \( h = 1.3r \).

\( \text{hullSurfaceArea}/1gLivingArea \) in a cylinder rotating about its long axis, the cylinder itself is all usable area, and the endcaps not (they form "walls"). The areas are:

\[ A_{\text{cyl}} = 2\pi rh \]  

(17)

\[ A_{\text{caps}} = 2(\pi r^2) \]  

(18)

Assuming \( h = 1.3r \), this yields:

\[ A_{\text{total}}/A_{\text{living}} = (A_{\text{cyl}} + A_{\text{caps}})/A_{\text{cyl}} = (2.6\pi r^2 + 2\pi r^2)/(2.6\pi r^2) = 1.77 \]  

(19)

This figure is used in this paper.